Michael Ruzhansky Ph.D.
Honorary Professor of Pure Mathematics

Imperial College London

31 March 2019

Report on the thesis by Sabitbek Bolys
"Hardy-Sobolev type inequalities on homogeneous groups and applications"
presented for a scientific degree Doctor of Philosophy (PhD).

This PhD thesis is devoted to investigating the research developments at the intersection of two subjects: functional inequalities, and the noncommutative analysis in the setting the stratified Lie groups (homogeneous Carnot groups).

The PhD thesis deals with four subjects in the study of subelliptic functional inequalities, namely, geometric functional inequalities, horizontal functional inequalities, functional inequalities with the sub-Laplacian fundamental solutions, and functional inequalities for general vector fields.

In the first subject, the geometric functional inequalities, he solved the conjecture regarding the natural weight in the geometric Hardy inequality. Also, he established the geometric Hardy-Sobolev inequality on the Heisenberg group and the Hardy inequality on starshaped sets on the Carnot groups, respectively. As a result, he has three submitted papers in this direction.
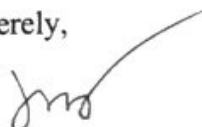
In the second subject, he generalised the previous results in this area and obtained further interesting results such as analogues of Hardy type inequalities with multiple singularities and many-particle Hardy type inequalities. Here he has published two papers in high-ranked journals.

In the third subject, he obtained the generalised weighted Lp-Hardy, Lp-Rellich, and Lp-Caffarelli-Kohn-Nirenberg type inequalities with boundary terms on stratified groups. As consequences, most of the Hardy type inequalities and the Heisenberg-Pauli-Weyl type uncertainty principles on the stratified groups have been also recovered. Here he published two papers in high-ranked journals.

In the final subject, which also opens up many topics for future research, he started working with general vector fields satisfying the Hörmander condition. Here he established the weighted Hardy and Rellich inequalities for general vector fields, and this was work accepted to the journal "Nonlinear Differential Equations and Applications".

In conclusion, this PhD thesis of Bolys Sabitbek represents a great deal of work. The results are well presented and their interpretation is at a high scientific level. The research it describes is of the international standard. This thesis is ready to be defended orally and certainly meets the requirements laid down for the degree of PhD in mathematics.

Sincerely,

Professor Michael Ruzhansky

Department of Mathematics
Imperial College London
180 Queen's Gate
London SW7 2AZ
United Kingdom

Telephone: +44 (0) 20 7594 8500
Fax: +44 (0) 20 7594 8517
Email: m.ruzhansky@imperial.ac.uk
URL: http://www.ma.ic.ac.uk/~ruzh/